14.1/14.3 Intro to Multivariable Functions and Partial Derivatives

Def'n: A function, *f*, of two variables is a rule that assigns a number for each input (x,y).

$$z = f(x, y).$$

In 3D:
(x,y) is the location on the xy-plane
z = f(x,y) = height above that point.

We sometimes write $f: \mathbb{R}^2 \to \mathbb{R}$.

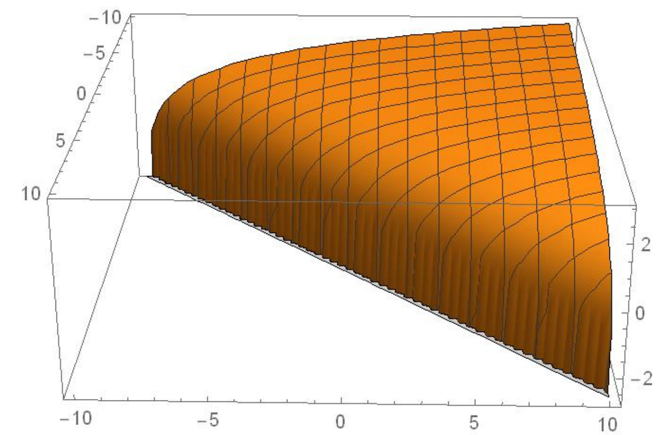
The set of allowable inputs is called the **domain**. Any question that asks "find the domain" is simply asking you if you know your functions well enough to understand when they are not defined.

Appears in Function	*Restriction*
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH ≠ 0
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	-1 ≤ BLAH ≤ 1
and other trig…	

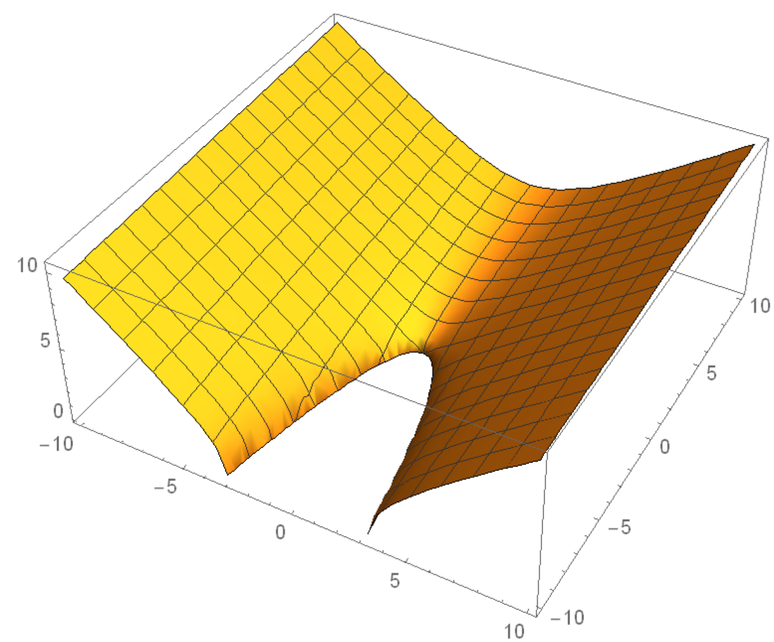
Examples:

Sketch the domain of

(1) $f(x, y) = \ln(y - x)$



(2) $g(x, y) = \sqrt{y + x^2}$

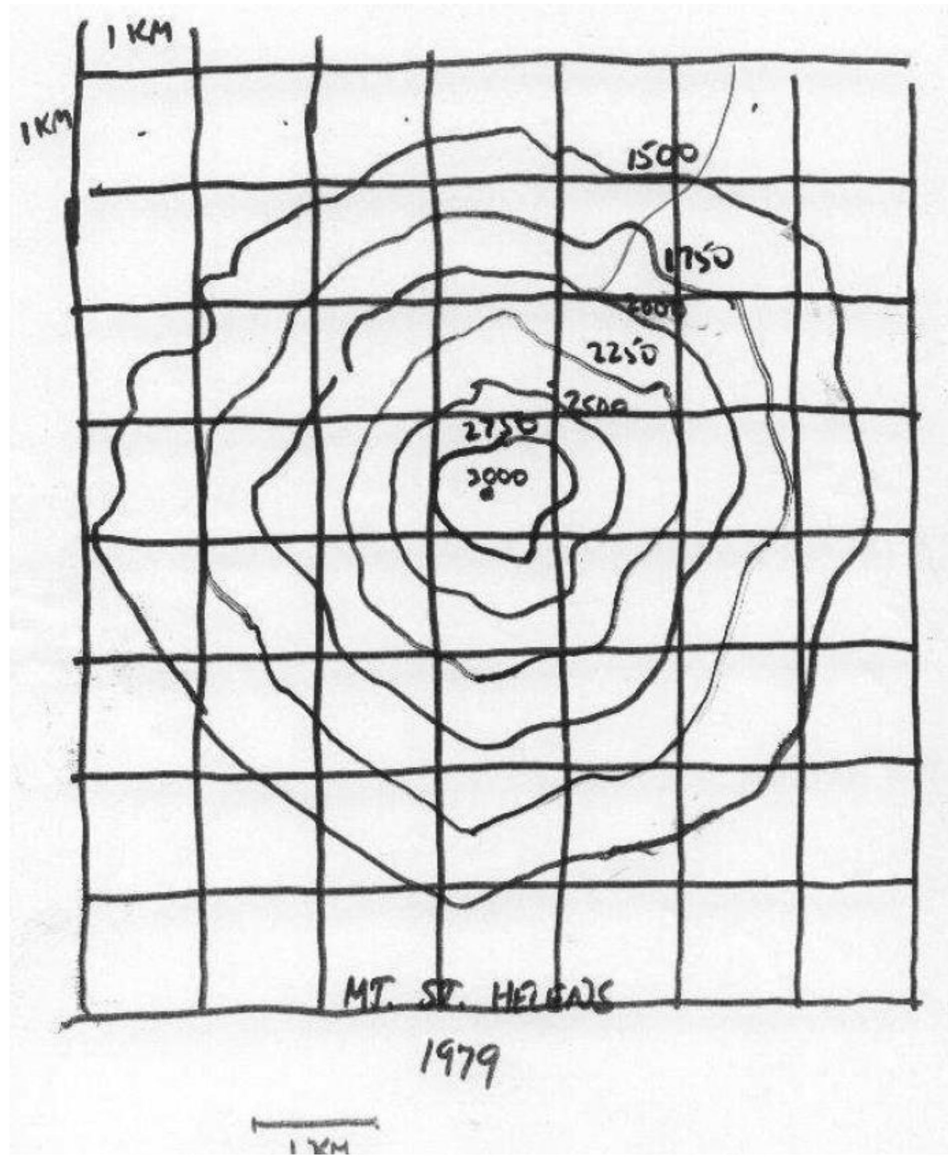


Visualizing Surfaces

The basic tool for graphing surfaces is **traces**. We typically look at traces given by fixed values of z (height) first.

We call these traces **level curves**, because each curve represents all the points at the same height (level) on the surface. A collection of level curves is called a **contour map** (or **elevation map**).

Contour Map (Elevation Map) of Mt. St. Helens from 1979 (before it erupted):

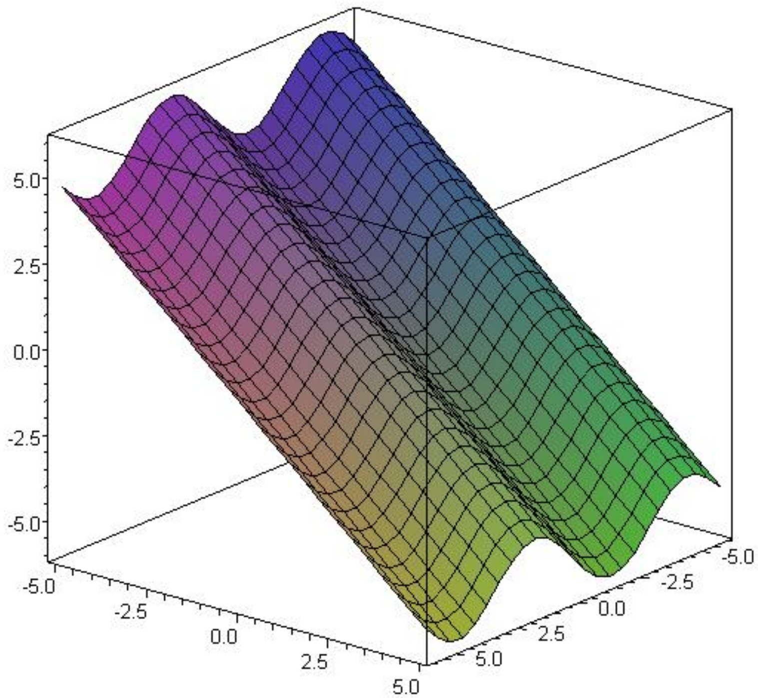


Examples:

1. Graph the level curves for z = -2, -1, 0, 1, and 2 for
$$z = f(x, y) = y - x$$

2. Graph level curves for
$$z = f(x, y) = \sin(x) - y$$

Graph of $z = \sin(x) - y$



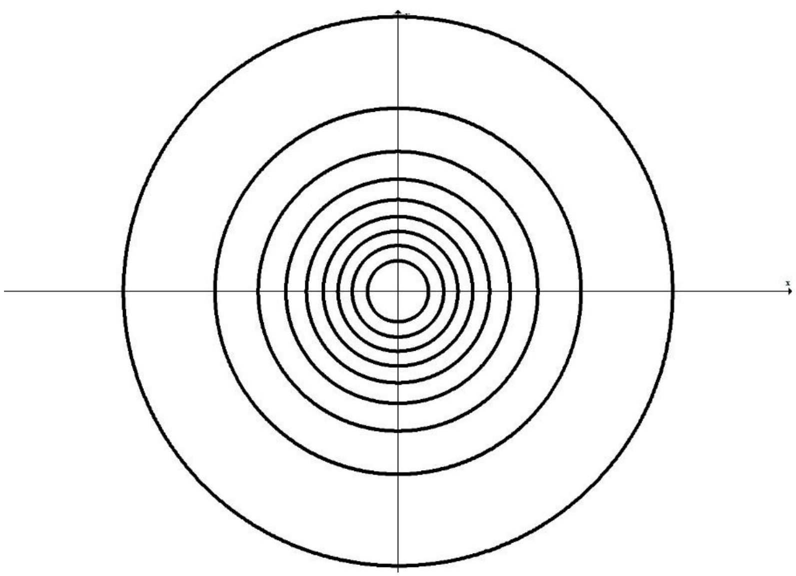
3. Graph level curves for

$$z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

Level Curves for

$$z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

at z = 1/10, 2/10, … , 9/10, 10/10



Graph of $z = f(x, y) = \frac{1}{1+x^2+y^2}$